Symbolic Model checking of Relative Safety LTL properties

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Outline

Motivation

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$$\mathcal{M} \models_{LTL} \varphi$$ ⚙ ⌛ ☹

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Doable for safetyLTL (assuming absense of deadlocks/livelocks)
Goal/Contribution: Generalize the approach for a larger fragment of LTL using relative safety

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Deadlock/livelock: A deadlock (livelock) state is a state from which no (fair) state is reachable

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Counterexample shape of $\alpha \rightarrow \varphi_{BR}$: $\underbrace{\overset{in \wedge t=p}{\bullet} \rightarrow \overset{t=p+2}{\bullet} \rightarrow \overset{t=p+5.01}{\bullet}}_{\text{Bad prefix}} \rightarrow \ldots$

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It would be nice to be able to verify this property using invariants and getting rid of deadlocks/livelocks!

Contributions

1. Reduce safetyLTL to invariant checking $+$ Block deadlocks
2. Extend safety LTL to invariant with relative safety to cover a larger fragment $\alpha_S \wedge \alpha_L \rightarrow \varphi$
3. Generalize (ii) blocking unfair counterexamples

Background

Properties classification

Properties classification

Safety property:

$P_S \subseteq \Sigma^\omega$ is a *safety property* iff $\forall \pi \in \Sigma^\omega$ s.t. $\pi \not\models P_S$, $\exists \pi_f \in Pref(\pi)$ s.t. $\forall \pi^\omega \in \Sigma^\omega : \pi_f \pi^\omega \not\models P_S$.

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Generic property:

$$P = P_S \cap P_L$$

LTL

Syntax: $\phi := \overbrace{\top \mid p \mid \phi \vee \phi \mid \neg\phi}^{\text{Propositional}} \mid \overbrace{\mathbf{X}\phi \mid \phi\mathbf{U}\phi}^{\text{future}} \mid \overbrace{\mathbf{Y}\phi \mid \phi\mathbf{S}\phi}^{\text{Past}}$

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Abbreviations: $\bot := \neg\top$ $\quad \mathbf{F}\phi := \top\mathbf{U}\phi$ $\quad \phi_1\mathbf{R}\phi_2 := \neg(\neg\phi_1\mathbf{U}\neg\phi_2)$ $\quad \mathbf{G}\phi := \bot\mathbf{R}\phi$
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Semantics (graphical repr):

$$\varphi\mathbf{U}\psi \quad \overset{\varphi}{\bullet} \to \overset{\varphi}{\ldots} \to \overset{\psi}{\bullet} \to \overset{\omega}{\ldots}$$

$$\mathbf{X}\varphi \quad \bullet \to \overset{\varphi}{\bullet} \to \overset{\omega}{\ldots}$$

$$\varphi\mathbf{S}\psi \quad \cdots \to \overset{\psi}{\bullet} \to \overset{\varphi}{\ldots} \to \overset{\varphi}{\bullet} \to \overset{\omega}{\ldots}$$

$$\mathbf{Y}\varphi \quad \cdots \to \overset{\varphi}{\bullet} \to \bullet \to \overset{\omega}{\ldots}$$

Safety fragments of LTL

Safety LTL (nnf): $\phi := \phi \vee \phi \mid \phi \wedge \phi \mid p \mid \underbrace{\neg p}_{\text{Neg on leaves}} \mid \mathbf{X}\phi \mid \underbrace{\phi \mathbf{R} \phi}_{\text{No until}} \mid \mathbf{Y}\phi \mid \phi\mathbf{S}\phi \mid \mathbf{Z}\phi \mid \phi\mathbf{T}\phi$

G α-past: $\phi := \mathbf{G}\phi_P \qquad \phi_P := p \mid \neg\phi_P \mid \phi_P \vee \phi_P \mid \mathbf{Y}\phi_P \mid \phi_P\mathbf{S}\phi_P$

Relation with safety:

$$Safety \cap LTL \equiv safetyLTL \equiv G\alpha\text{-past}$$[Chang, Manna Pnuelli 92]

Relative safety[Henzinger92]

Let P and A be two properties. P is safety relative to A iff

$$\forall \pi \in A \text{ s.t. } \pi \notin P, \exists \pi_f \in Pref(\pi) \text{ s.t.}$$
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Notable examples:

- φ_S is safety relative to $\top$.
- $\mathbf{G}p \rightarrow \mathbf{G}q$ is safety relative to $\mathbf{G}p$.
- Bounded response is safety relative to non-zenoness and weak monotonicity.
- $p\mathbf{U}q$ is safety relative to $\mathbf{F}q$

Contributions

Steps

1. Construct a safetyLTL model checking procedure reducing to invariant checking
2. Extend the approach to the fragment: $\underbrace{\alpha_S}_{\text{Safety property}} \wedge \underbrace{\alpha_L}_{\text{Liveness property}} \rightarrow \underbrace{\varphi}_{\text{safetyLTL}}$ (complete with assumptions)
3. Generalize the approach to generic $\underbrace{\alpha}_{\text{LTL}} \rightarrow \underbrace{\varphi}_{\text{safetyLTL}}$ using an iterative approach to block unfair counterexamples

Note: Extension can be done because $\alpha \rightarrow \varphi$ safety relative to α.

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If $\mathcal{M}$ is deadlock free, then

$$\mathcal{M} \models \phi_S \Leftrightarrow \mathcal{M} \models_{INVAR} \phi_S$$

Basic algorithm (no loop)

Check: $\underbrace{\mathcal{M}}_{\langle V, \mathcal{I}, \mathcal{T}, \mathcal{F} \rangle \text{fair STS}} \models \underbrace{\alpha_S}_{\text{safety}} \wedge \underbrace{\alpha_L}_{\text{liveness}} \rightarrow \underbrace{\varphi}_{\text{safetyLTL}}$

$\boxed{\mathcal{M}' := \mathcal{M} \times \mathcal{M}_{\alpha_S}}$

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$\downarrow$

$\mathcal{M}' \models_{INVAR} \varphi$

$\downarrow$ Yes

VALID

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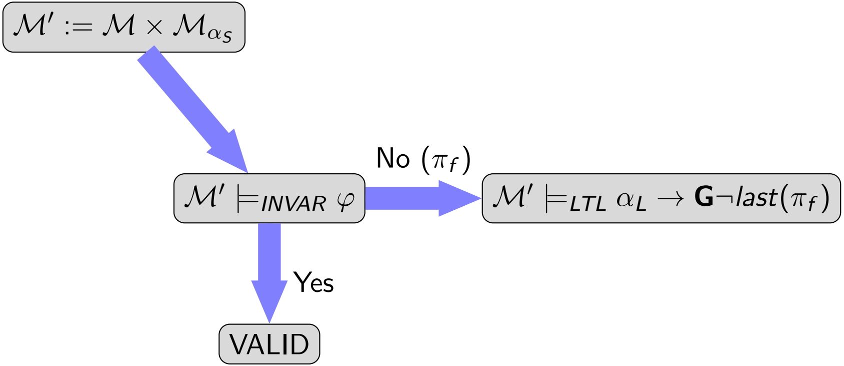

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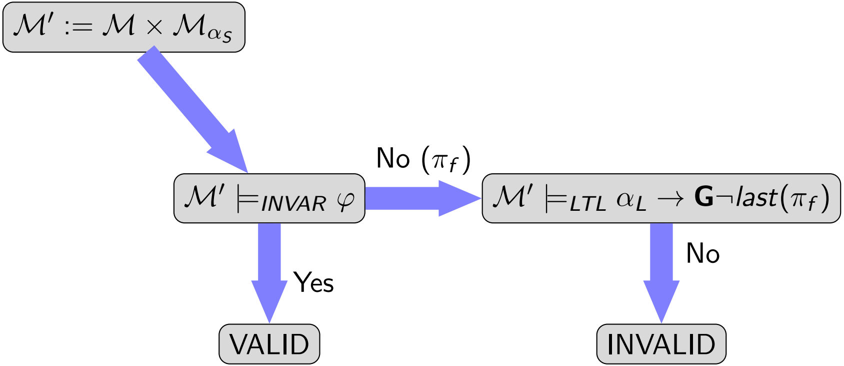

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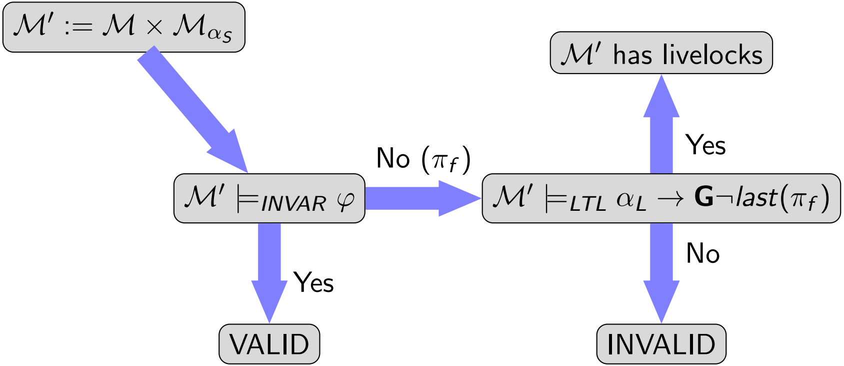

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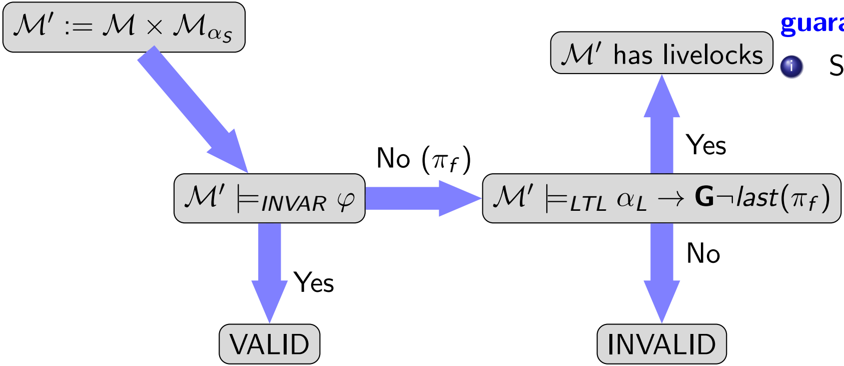

guarantees:

- Soundness Yes

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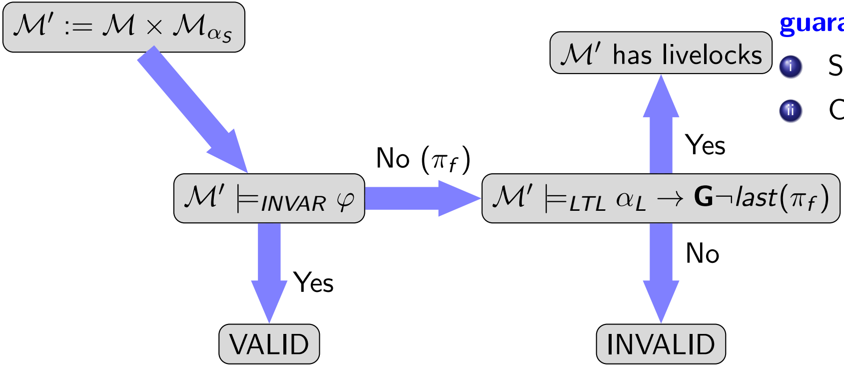

guarantees:

1. Soundness Yes
2. Completeness No

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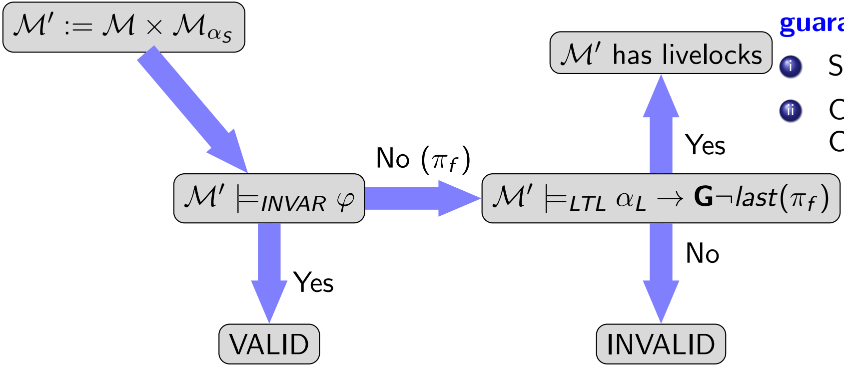
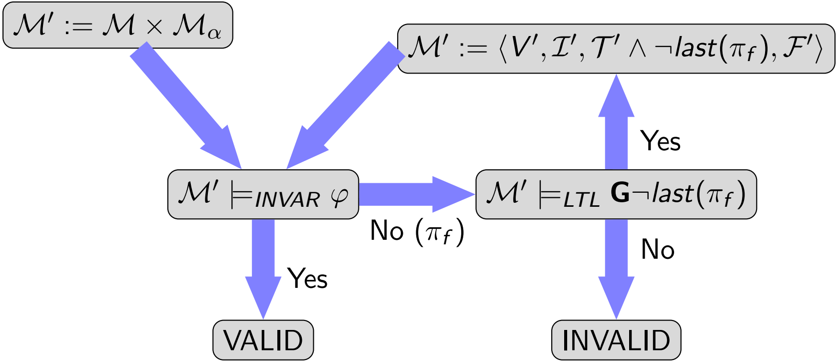


guarantees:

1. Soundness — Yes
2. Completeness — No
   Complete assuming:
   1. Invariant and LTL checks terminate,
   2. $\alpha_S := \mathbf{G}\alpha_P$ (construction does not introduce deadlocks),
   3. $\mathcal{M}$ is live w.r.t. $\alpha_S \wedge \alpha_L$

Check: $\underbrace{\mathcal{M}}_{\langle V,\mathcal{I},\mathcal{T},\mathcal{F}\rangle \text{fair STS}} \models \underbrace{\alpha}_{\text{LTL}} \rightarrow \underbrace{\varphi}_{\text{safetyLTL}}$



Iterative algorithm

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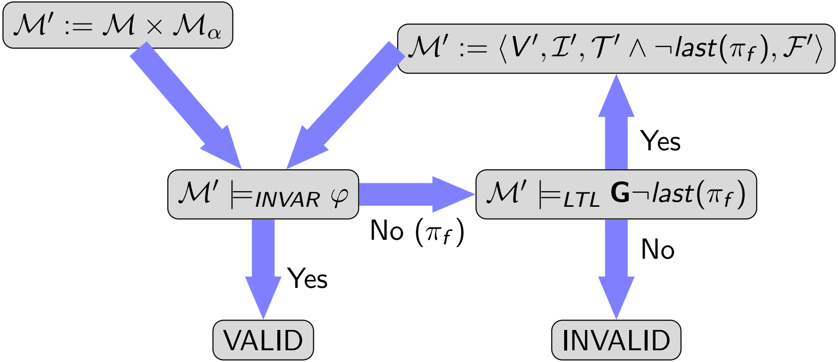


Complete for finite state STS

Optimization: extending safetyLTL verification with lookahead

Idea:

- If $\mathcal{M}'$ has *livelocks*, multiple iterations are required
- Computing steps ahead for counterexamples can rule out deadlock states.

Discard:

$$\underbrace{\bullet \to \bullet \to \bullet}_{\text{Bad prefix } \pi_f} \to \underbrace{X}_{\text{Deadlock}}$$

Consider:

$$\underbrace{\bullet \to \bullet \to \bullet}_{\text{Bad prefix } \pi'_f} \to \overset{la=1}{\bullet} \to \underbrace{\dots}_{n-2 \text{ steps}} \to \overset{la=n}{\bullet}$$

Procedure:

- When the original invariant is falsified start incrementing *la*
- New INVAR: $la < n$

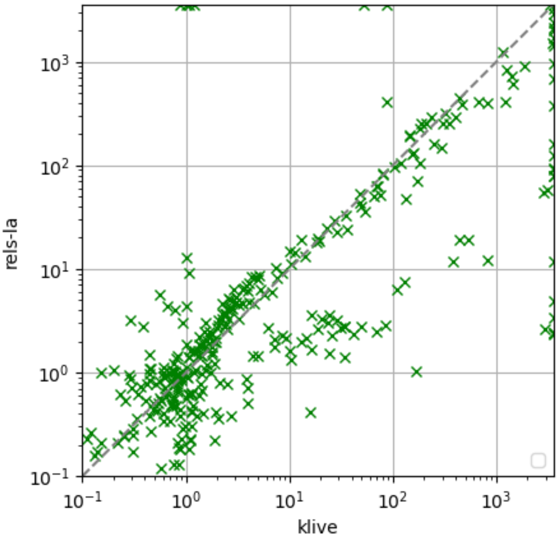
Experimental evaluation

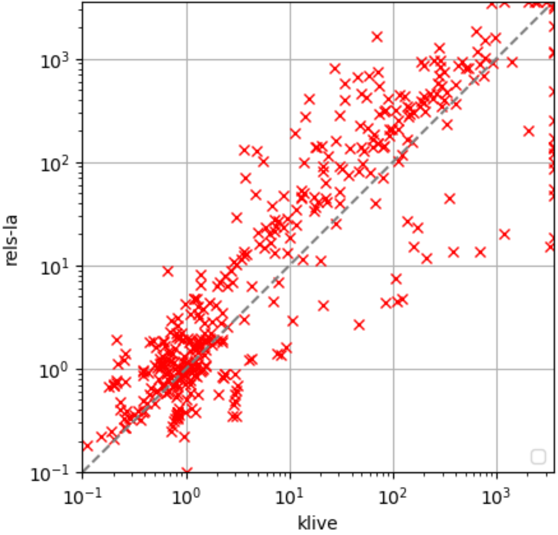
- Implemented inside nuXmv symbolic model checker on top of SMT based infinite state invariant checking.
- LTL check using K-liveness with IC3
- Invariant checking done with IC3
- Comparison with k-liveness[K. Claessen and N. Sörensson 2012], liveness to safety[A. Biere, C. Artho, and V. Schuppan 2002] (adapted for infinite state).

Models:

- A/G contracts (e.g. Wheel Brake System)
- Bounded response (infinite state)
- Asynchronous systems with fair scheduling $\bigwedge_i \mathbf{GF} run_i \rightarrow \varphi$
- NuSMV models (finite state)
- nuXmv models (infinite state)

Comparison with k-liveness





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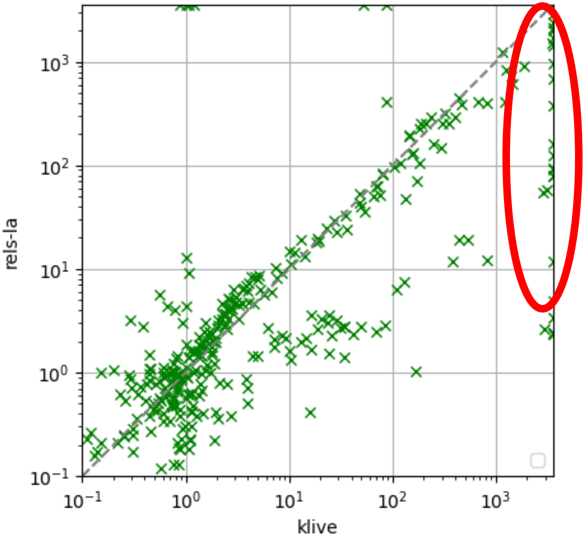

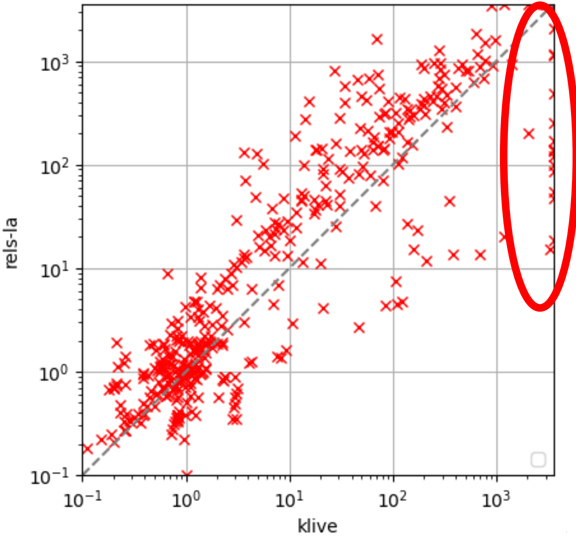

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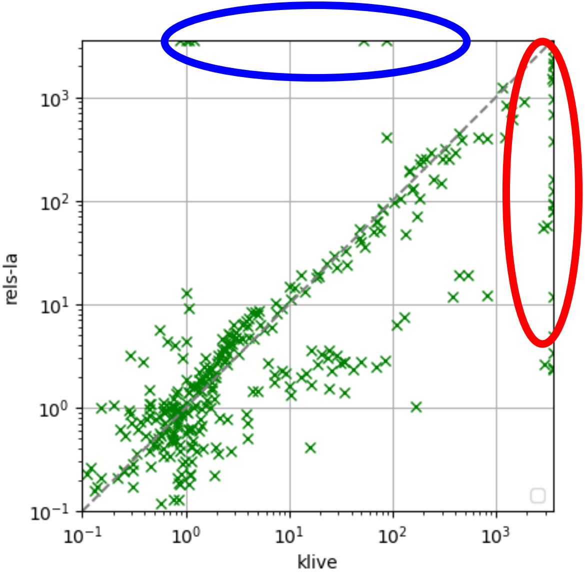
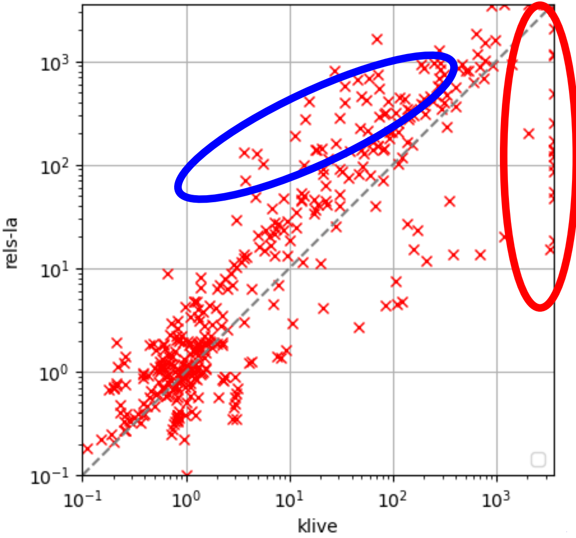
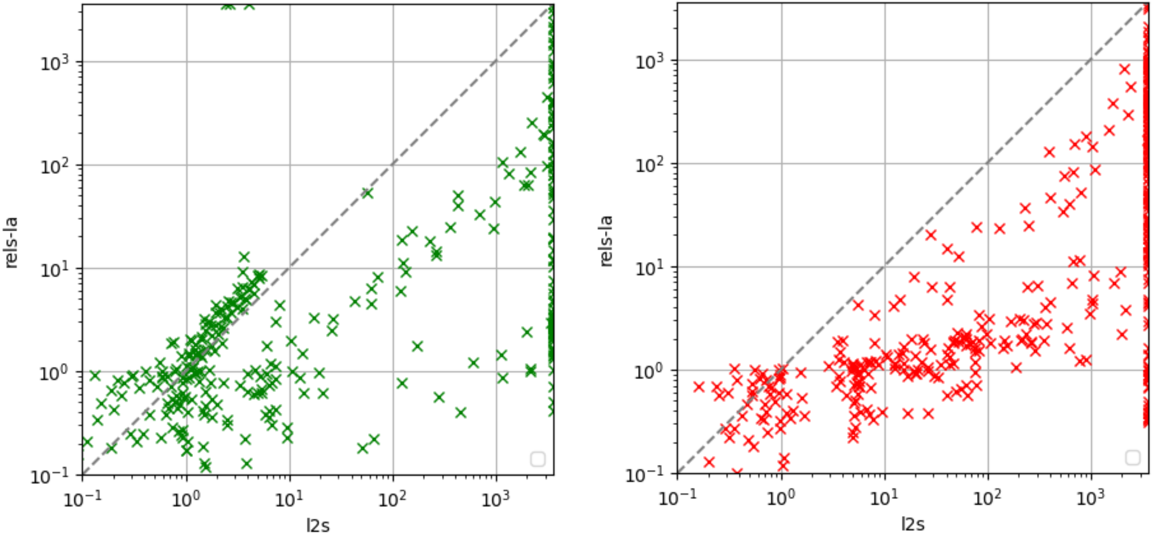




Comparison with liveness to safety



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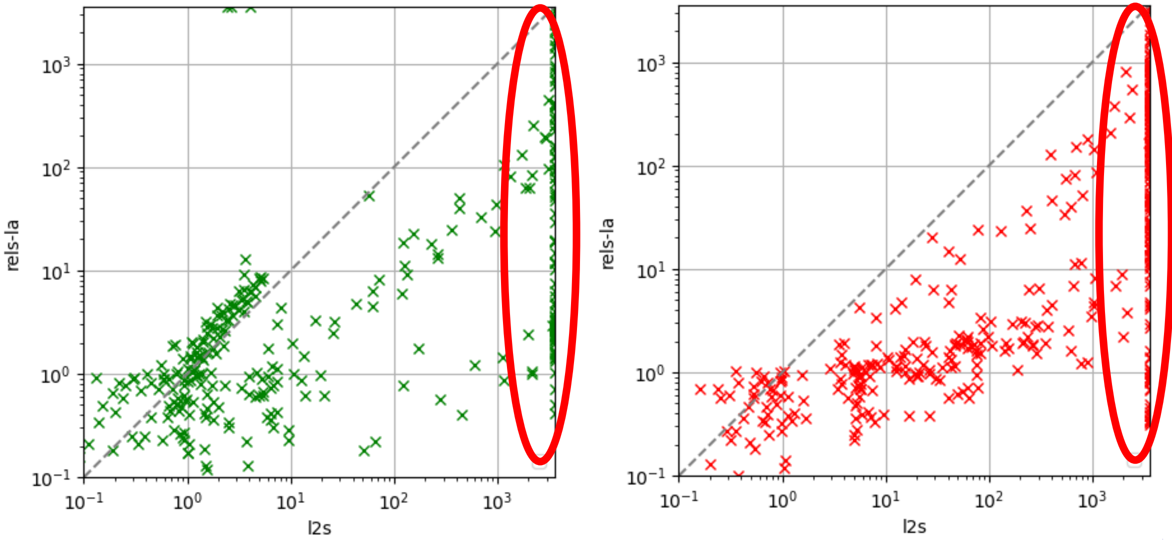

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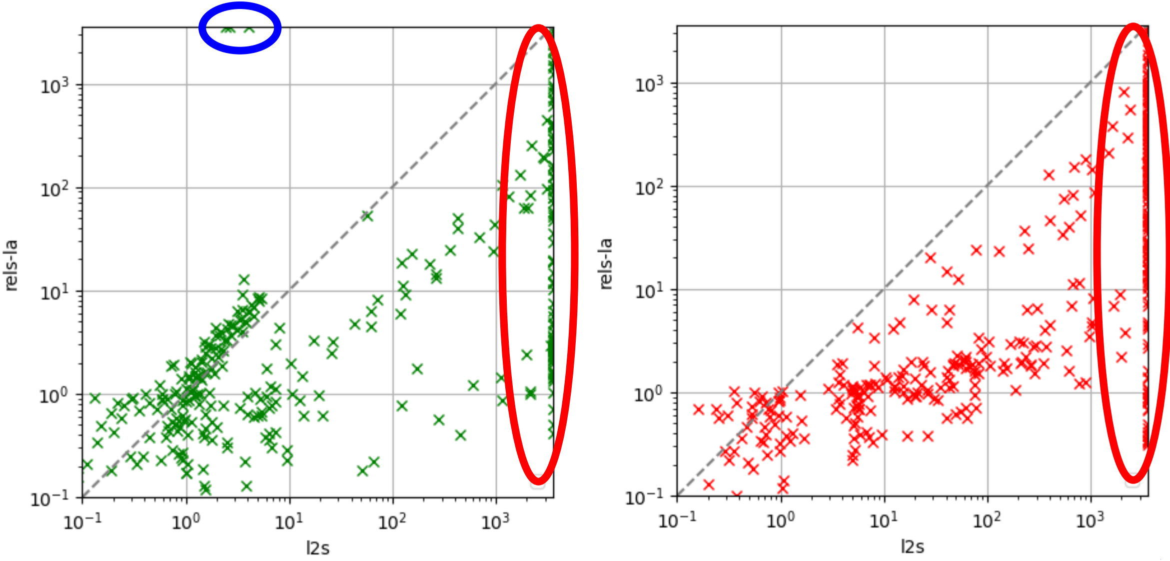
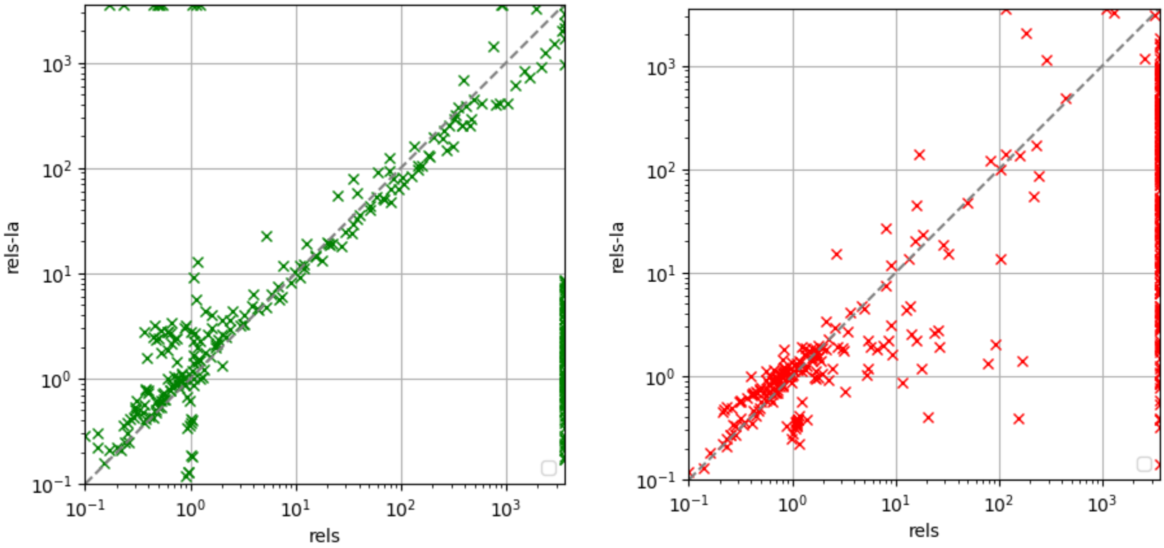


Impact of lookahead construction



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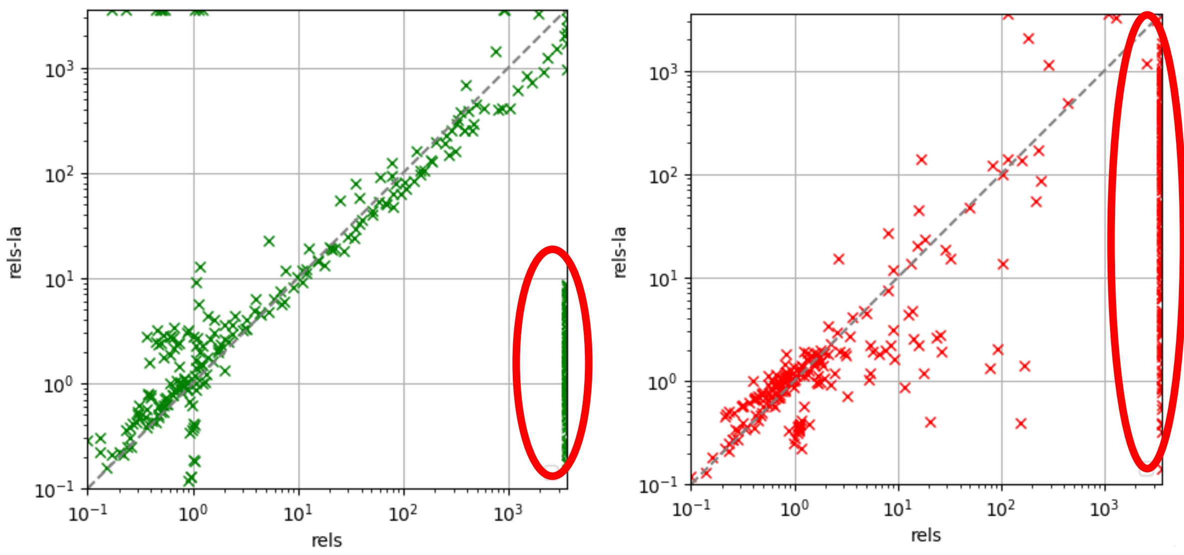

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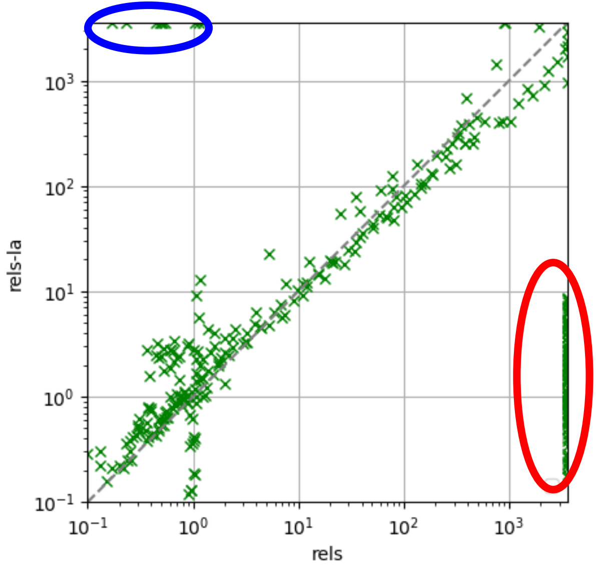

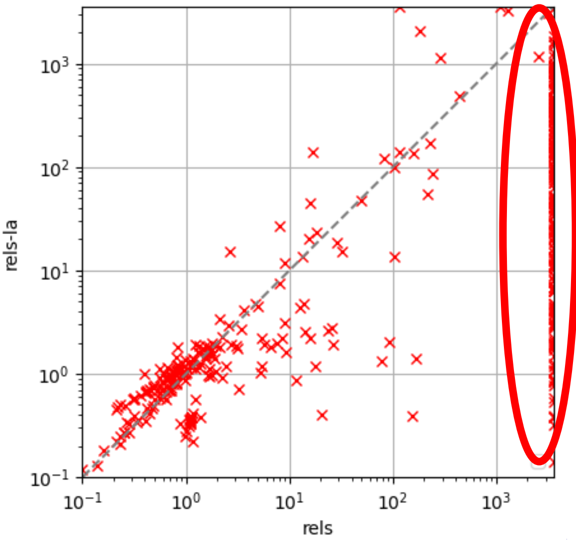

Conclusion

Concluding remarks

Considerations:

1. Deadlocks and livelocks are the main obstacle, many times the LTL to automata construction introduces the deadlocks with prophecy variables ($v_{\mathbf{X}\beta}$).
2. Providing a finite lookahead computation is sufficient to rule out many spurious counterexamples.
3. There are rooms for improvements (next slide)

Future directions

Improvements of the algorithm:

- Counterexample generalization exploiting k-liveness (using inductive invariants)
- Consider using temporal testers
- Extend with lockstep with BMC (as for k-liveness)
- Exploit SMT solver incrementality

Applications of the algorithm:

- Extend the fragment such that φ can be non-safety
- Targetting continuous time
- Apply to contract-based verification compositionally where A/G are formulae

Questions?

Appendix

LTL model checking

Standard LTL model checking:

$$\mathcal{M} \models_{LTL} \phi \Leftrightarrow \mathcal{M} \times \mathcal{M}_{\neg\phi} \models \neg \bigwedge_{f_i \in \mathcal{F}_{\neg\phi}} \mathbf{GF} f_i$$

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&\quad \{ v_{\mathbf{X}(\phi_1 \mathbf{U} \phi_2)} \mid \phi_1 \mathbf{U} \phi_2 \in Sub(\phi) \} \\
\mathcal{I}_\phi &:= Enc(\phi)
\end{aligned}$$

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1. Introduce prophecy variables for temporal operators.
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5. Enforce fairness for until

Informative prefix[Kupferman, Vardi 2001]

Definition

Let ψ be an LTL formula in negative normal form, $Sub(\psi)$ be the set of sub-formulas of ψ and let π be a finite path of length n over the language of ψ. We say that π is *informative* for ψ iff there exists a mapping $L : \{0, \ldots, n\} \to 2^{Sub(\neg\psi)}$ such that:

1. $\neg\psi \in L(0)$.
2. $L(n) = \emptyset$.
3. For all $0 \leq i < n$, forall $\varphi \in L(i)$:
   - If φ is propositional, $\pi, i \models \varphi$.
   - If $\varphi = \varphi_1 \vee \varphi_2$, $\varphi_1 \in L(i)$ or $\varphi_2 \in L(i)$.
   - If $\varphi = \varphi_1 \wedge \varphi_2$, $\varphi_1 \in L(i)$ and $\varphi_2 \in L(i)$.
   - If $\varphi = \mathbf{X}\varphi_1$, $\varphi_1 \in L(i+1)$
   - If $\varphi = \varphi_1 \mathbf{U} \varphi_2$, $\varphi_2 \in L(i)$ or $[\varphi_1 \in L(i)$ and $\varphi_1 \mathbf{U} \varphi_2 \in L(i+1)]$.
   - If $\varphi = \varphi_1 \mathbf{R} \varphi_2$, $\varphi_2 \in L(i)$ and $[\varphi_1 \in L(i)$ or $\varphi_1 \mathbf{R} \varphi_2 \in L(i+1)]$.

Safety LTL model checking

High level idea:

- Rewrite $\neg\phi$ in *nnf*
- Construct STS of $\neg\phi$ (similar to LTL2SMV)
- Compute invariant $INV_\phi := \neg(\bigwedge_{v_{\mathbf{X}\beta}} \neg v_{\mathbf{X}\beta})$

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$$I_{\neg\phi} = Enc(\neg\phi) \wedge \bigwedge_{v_{\mathbf{Y}\beta} \in V_{\neg\phi}} \neg v_{\mathbf{Y}\beta}$$

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$Enc(\varphi)$:

- $Enc(\phi_1 \wedge \phi_2) = Enc(\phi_1) \wedge Enc(\phi_2)$, $Enc(\phi_1 \vee \phi_2) = Enc(\phi_1) \vee Enc(\phi_2)$
- $Enc(\neg\phi_1) = \neg Enc(\phi_1)$
- $Enc(\mathbf{X}\phi_1) = v_{\mathbf{X}\phi_1}$
- $Enc(\phi_1 \mathbf{U} \phi_2) = Enc(\phi_2) \vee (Enc(\phi_1) \wedge v_{\mathbf{X}(\phi_1 \mathbf{U} \phi_2)})$

Extended motivating example

Bounded response: $\varphi := \mathbf{G}(in \wedge t = p \rightarrow \mathbf{F}(t \leq p + 5 \wedge out))$

$$\alpha := \underbrace{\mathbf{G}(t' \geq t)}_{\text{Weak monotonicity}} \wedge \underbrace{\mathbf{GF}(t' - t \geq \epsilon)}_{\text{non-zenoness}}$$

Assuming α, bounded response can be reduced to $(\alpha \rightarrow (\varphi \leftrightarrow \varphi_S))$

$$\varphi_S := \mathbf{G}(in \wedge t = p \rightarrow out \mathbf{R} t \leq p + 5)$$

Counterexample shape of φ_S: $\underbrace{\overset{in \wedge t = p}{\bullet} \rightarrow \overset{t = p + 2}{\bullet} \rightarrow \overset{t = p + 5.01}{\bullet}}_{\text{Bad prefix}} \rightarrow \ldots$

Any finite counterexample of φ_S that can be extended to infinity is a counterexample of $\alpha \rightarrow \varphi$